Mean Period of Fluctuations Near the Wall in Turbulent Flows

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In the field of fluid mechanics, the development of the Navier-Stokes equations has resulted in a significant union of the theoretical and the practical when the flow under consideration is of a viscous nature. Unfortunately in the case of turbulent flow, an adequate description of the basic processes is not known (Batchelor, 1953; Hinze, 1959; Townsend, 1956; Kraichnan, 1962; Lee, 1965), and it appears unlikely that a perfectly general description of turbulent flow can be developed. For the all-important case of inhomogeneous, anisotropic turbulence as found in a conduit, purely theoretical approaches have led to little of much practical use. Thus, it is permissible to limit the scope of an attack on the turbulent-flow problem to approaches which yield quantitative relationships concerning only part of the flow field. In the past several years many investigators have focused their attention on the viscous sublayer region of turbulent shear flow, which, as has been by now well established, can in no sense be regarded as dynamically time independent (Meek and Baer, 1970a). Rather, flow in the sublayer region is composed of a sequence of events having a statistically defined average periodicity. In this note critical examination of the sum total of the measurements of the mean sublayer period is made. It is concluded that the mean period scales with the friction velocity and kinematic viscosity rather than with the outer flow parameters u_∞ and δ.

Variables are rendered dimensionless in the usual "law of the wall" formalism; that is, by combination with the kinematic viscosity ν and the friction velocity u_* (dimensionless lengths are then $y \rightarrow y^+ = yu_*/\nu$, for example). It seems most natural to make time dimensionless by first going to the characteristic length $(\nu t)^{1/2}$ and then combining with u_* and ν in the usual manner; thus $t \rightarrow t^+ = (\nu t)^{1/2}\, u_*/\nu \equiv (t/\nu)^{1/2} u_*$. This dimensionless time variable, which is just the square root of that used by others, is mathematically more convenient in periodic viscous sublayer analyses and leads to a natural simplicity in the important results; for example, the dimensionless sublayer thickness is proportional to the dimensionless sublayer development time (Meek, 1968). The Reynolds number is that usually used for tube flow $D\langle u\rangle/\nu$ where D is the tube diameter and $\langle u\rangle$ is the bulk velocity. For boundary layer flows, the appropriate Reynolds number commensurate with this convention is $2\delta\langle u\rangle/\nu$ where δ is the turbulent boundary layer thickness. Where necessary, Reynolds numbers based on the momentum thickness have been converted using a 1/7 power law velocity profile.

The mean viscous sublayer period data determined by various investigators either by autocorrelation of the signal from a sensor at or very near the wall or by visual observation of the frequency of major sublayer disturbances are summarized in Figure 1. The boundary-layer probe data of Kim et al., (1968); Rao et al., (1969); and Tu and Willmarth (1966) have been discussed recently by Laufer and Badri Narayanan (1971). Schraub's (Schraub and Kline, 1965; Kline et al., 1967; Schraub et al., 1965) burst rate data are indicated by the doubled line in Figure 1. In those experiments, a dye was injected through a slot transverse to the flow direction, and the rate at which slow-moving, wall-adjacent fluid was ejected into the mainstream was determined. The longitudinal extent of a sublayer element (Meek, 1968; Mitchell, 1965; Mitchell and Hanratty, 1966) is such that only one sublayer-patch length was included in the area over which the burst count was made. Then $F = \dfrac{1}{\Delta_x T}$ where F is burst rate per unit length transverse to the flow and Δ_x is the transverse sublayer extent (Meek, 1968). From experimental data $\Delta_x{}^+ \simeq 50$ (Meek, 1968; Mitchell, 1965; Runstadler, et al., 1963) so that Schraub's data in conjunction with those of Runstadler (1963) yield the result that $T^+ \simeq 14$. The visual observations of Corino and Brodkey (1969) as presented in Figure 8 of that paper are also plotted in Figure 1 where the average number of disturbances per second at a point on the wall have been converted to dimensionless mean periods. Armistead and Keyes (1968) have investigated fluctuations in heat transfer at the wall and the period determined by autocorrelation for the one case where a (very weak) maximum was observed is also presented in Figure 1.

The present author has measured fluctuations of the viscous sublayer by use of a boundary-layer probe, a hot wire pressure transducer, and painted-on platinum film resistance thermometers (Meek, 1968). The boundary layer probe was of conventional commercial (Thermo-Systems) design. The pressure transducer (Thermo-Systems Model 1410) was of the kind first proposed by Remenyik and Kovasznay (1962), while the platinum film wall temperature sensors have been described elsewhere (Meek, 1968; Meek and Baer, 1972b). The wall pressure sensor had a sensitivity of 10^{-5} lb./sq.in. and flat frequency response to 200 Hz. The dimensionless mean periods as determined from autocorrelation of the transducer signals, after linearization in the hot wire cases, are presented in Figure 1. Finally, Mitchell's (1965) mass transfer spectra indicate the existence of a rather broad maximum at $fd/\langle u\rangle \sim 0.05$ for Reynolds numbers of 1 to 2 $(10)^4$ corresponding to T^+ of approximately 30.

As mentioned previously, Laufer and Badri Narayanan (1971) have discussed much of the boundary layer probe data previously. They find they are well represented by $T^+ = 0.28\, N_{Re}{}^{.375}$ in terms of the present variables, and that in fact, the group Tu_∞/δ is approximately constant at 5. This result, that processes near the wall scale with the outer flow parameters, was shown to be expected from the $T^+(N_{Re})$ relation by use of some simple arguments. However, as can be seen from Figure 1, this relation does

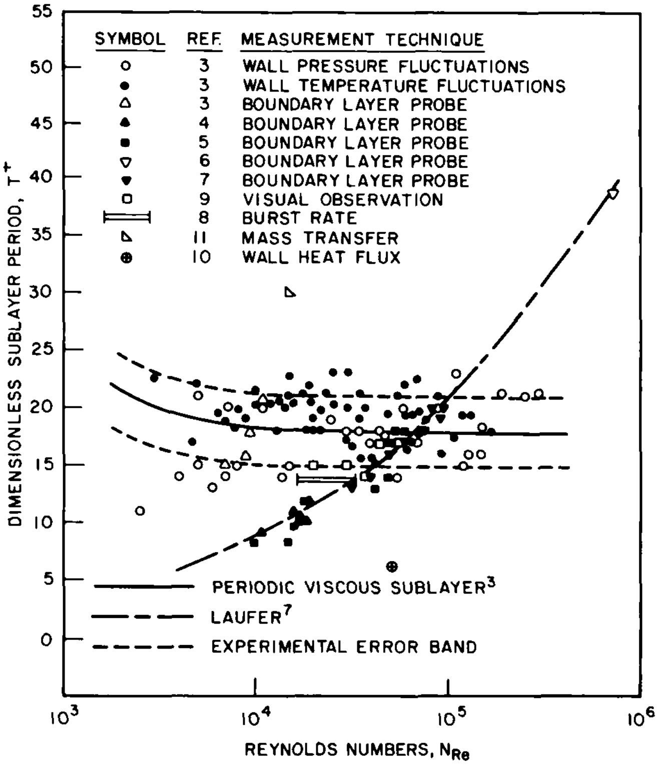

Fig. 1. Dimensionless mean sublayer period versus Reynolds number.

not fit the bulk of the data.

Meek and Baer (1970a) have recently redeveloped the periodic viscous sublayer model of Einstein and Li (1956) in a self-consistent manner involving no arbitrary parameters. The $T^+(N_{Re})$ relation calculated from the model is also given in Figure 1. For Reynolds numbers greater than $\sim 10^4$ the model prediction is that T^+ is essentially constant at 18. Clearly most of the data are in basic agreement with this conclusion. In fact most of the author's data are in agreement within the expected experimental uncertainty and similar uncertainties are to be expected in the other data. As noted, the peak in the data of Mitchell (1965) was rather ill-defined, and Armistead and Keyes (1968) correlation was also rather weak. In fact in the latter work the available time delays for autocorrelation were not great enough to have detected the predicted period.

While the wall temperature sensors are nondisrupting, the boundary layer probe and the wall pressure transducer are not. The orifice in the wall for the pressure transducer is 0.2 cm and the boundary layer probe used by the author was of a similar lateral extent. In the air flow experiments reported, the maximum sublayer thickness and transverse patch size are only 0.1 to 1 cm (Meek, 1968) so that the presence of the probe may have a significant effect on the sublayer behavior. Of even greater potential importance is the recent work by Perry and Morrison (1971) on hot-wire anemometer filament vibrations which could produce serious errors. More extensive measurements utilizing a variety of observation techniques and encompassing a more extensive range of N_{Re} are clearly needed.

However, we have considerable confidence in the model predicted viscous sublayer period based on the results of heat (or mass) transfer calculations (Meek and Baer, 1972a). No adjustable parameters are involved and heat transfer coefficients in substantial agreement with experiment (in most cases within a few %) can be calculated for a large range of Prandtl numbers ($10^{-2} < N_{Pr} < 10^{+2}$). It is apparent that the computed Stanton number will decrease as the assumed period increases. (This is simply because the instantaneous flux is a monotonically decreasing function of time.) In fact to a first approximation, $N_{St} \propto 1/T^+$ (Meek and Baer, 1970b). This implies then that the sublayer development or residence time can not be too different from the model predicted value.

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